A PHS Tour in Audio/Acoustics

Applications in Virtual Reality, Control and Health

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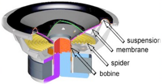
Spring School on Theory and Applications of Port-Hamiltonian Systems
31 March 2019 – 5 April 2019
Naraueninsel (Chiemsee)



Context and motivation

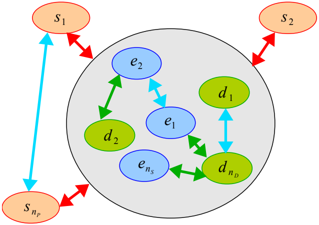
1. **Model input-output multi-physics systems** for sound and musical applications:
   - Phenomena: mechanical, acoustic, electronic, magnetic, etc
   - Realism: nonlinearities, non ideal dissipations, etc
2. **Satisfy fundamental physical properties**:
   - causality, stability, passivity and more precisely ...
   - the **power balance** structured into conservative/dissipative/source parts
   - other natural invariants and symmetries (if any)
3. **Simulate such systems and preserve these properties** in the discrete time domain *(+accuracy+sound quality/Shannon-Nyquist principle)*
4. **Design code generators** from netlists for real-time applications
5. **Design correctors and controllers** to reach target behaviours

Outline

1. Context
2. **Framework:** basics, recalls and tools
3. **Analog electronics** and electro-acoustics
4. **Mechanics:** nonlinear damped vibrations
5. **Control** applications in acoustics
6. **Voice:** a minimal model to analyze self-oscillations
7. Conclusion

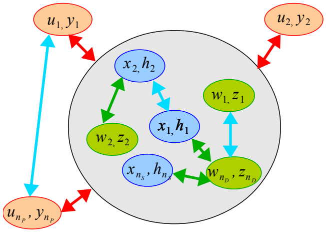
Outline

A physical system is made of. . .



- Energy-storing components: *(energy)*
  $E = \sum_{n=1}^{N} e_n \geq 0$
- Dissipative components: *(dissipated power)*
  $Q = \sum_{m=1}^{M} d_m \geq 0$
- External sources: *(external power)*
  $P_{\mathrm{ext}} = \sum_{p=1}^{P} s_p$
- Conservative connections *(power balance)*
  $\frac{dE}{dt} = P_{\mathrm{ext}} - Q$

A physical system is made of...



- Energy-storing components: *(energy)*
  $E = H(\mathbf{x}) = \sum_{n=1}^{N} H_n(x_n) \geq 0$
- Dissipative components: *(dissipated power)*
  $Q = \mathbf{z}(\mathbf{w})^T \mathbf{w} = \sum_{m=1}^{M} z_m(w_m)\, w_m \geq 0$
  (effort × flow : force × velocity, voltage × current, etc)
- External sources: *(external power)*
  $P_{\text{ext}} = \mathbf{u}^T \mathbf{y} = \sum_{p=1}^{P} u_p y_p$
- Conservative connections *(power balance)*
  $\nabla H(\mathbf{x})^T \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} + \mathbf{z}(\mathbf{w})^T \mathbf{w} - \mathbf{u}^T \mathbf{y} = 0$

Port-Hamiltonian Formulation — Power balance

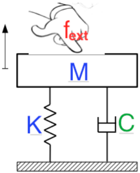
$$\underbrace{\begin{pmatrix} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} \\ \hline \mathbf{w} \\ \hline -\mathbf{y} \end{pmatrix}}_{B} = S \cdot \underbrace{\begin{pmatrix} \nabla H(\mathbf{x}) \\ \hline \mathbf{z}(\mathbf{w}) \\ \hline \mathbf{u} \end{pmatrix}}_{A}$$

$$\boxed{\begin{aligned} 0 &= A^T B \\ &= A^T S A \end{aligned}}$$

if $S = -S^T$

A: components constitutive laws & external actions,
S: interconnections between flows and efforts.

Example: damped mechanical oscillator



$\mathbf{x} = \begin{pmatrix} p \\ q \end{pmatrix}$	*momentum* *elongation*	$H(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T \begin{pmatrix} M^{-1} & 0 \\ 0 & K \end{pmatrix} \mathbf{x}$	*kinetic* *potential*
$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix}$	*inertia force* *spring velocity*	$\nabla H(\mathbf{x}) = \begin{pmatrix} p/M \\ K\,q \end{pmatrix}$	*mass velocity* *spring force*
$\mathbf{w} = v_{\mathrm{d}}$	*dashpot velocity*	$\mathbf{z}(\mathbf{w}) = Cv_{\mathrm{d}}$	*damping force*
$\mathbf{y} = v_{\mathrm{ext}}$	*external velocity*	$\mathbf{u} = f_{\mathrm{ext}}$	*external force*

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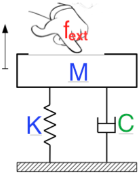

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$$\begin{pmatrix} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} \\ \hline \mathbf{w} \\ \hline -\mathbf{y} \end{pmatrix} = \underbrace{\left(\begin{array}{cc|c|c} 0 & -1 & -1 & +1 \\ +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \\ \hline -1 & 0 & 0 & 0 \end{array}\right)}_{S=-S^T} \cdot \begin{pmatrix} \nabla H(\mathbf{x}) \\ \hline \mathbf{z}(\mathbf{w}) \\ \hline \mathbf{u} \end{pmatrix}$$

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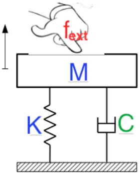

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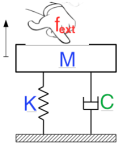
$$
\begin{aligned}
\dot{p} &= f_{\mathrm{ext}} - K\,q - C v_{\mathrm{d}} \\
\dot{q} &= p/M \\
v_{\mathrm{d}} &= p/M \\
v_{\mathrm{ext}} &= p/M
\end{aligned}
\quad \longleftarrow \quad
\begin{pmatrix} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} \\ \hline \mathbf{w} \\ \hline -\mathbf{y} \end{pmatrix} = \underbrace{\left(\begin{array}{cc|c|c} 0 & -1 & -1 & +1 \\ +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \\ \hline -1 & 0 & 0 & 0 \end{array}\right)}_{S=-S^T} \cdot \begin{pmatrix} \nabla H(\mathbf{x}) \\ \hline \mathbf{z}(\mathbf{w}) \\ \hline \mathbf{u} \end{pmatrix}
$$

Some variations



$$\begin{pmatrix} F_M \\ v_K \\ \hline v_C \\ \hline -v_{\mathrm{ext}} \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & -1 & -1 & +1 \\ +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \\ \hline -1 & 0 & 0 & 0 \end{array}\right) \cdot \begin{pmatrix} v_M \\ F_K \\ \hline F_C \\ \hline F_{\mathrm{ext}} \end{pmatrix}$$

Hamiltonian systems (conservative, autonomous)

$$\begin{pmatrix} F_M \\ v_K \\ \hline . \\ \hline . \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & -1 & . & . \\ +1 & 0 & . & . \\ \hline . & . & . & . \\ \hline . & . & . & . \end{array}\right) \cdot \begin{pmatrix} v_M \\ F_K \\ \hline . \\ \hline . \end{pmatrix}$$

"Mass+Damper+Excitation" (spring removed)

$$\begin{pmatrix} F_M \\ \hline . \\ \hline v_C \\ \hline -v_{\mathrm{ext}} \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & . & -1 & +1 \\ \hline . & . & . & . \\ \hline +1 & . & 0 & 0 \\ \hline -1 & . & 0 & 0 \end{array}\right) \cdot \begin{pmatrix} v_M \\ \hline . \\ \hline F_C \\ \hline F_{\mathrm{ext}} \end{pmatrix}$$

"Mass+Excitation"

$$\begin{pmatrix} F_M \\ \hline . \\ \hline . \\ \hline -v_{\mathrm{ext}} \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & . & . & +1 \\ \hline . & . & . & . \\ \hline . & . & . & . \\ \hline -1 & . & . & 0 \end{array}\right) \cdot \begin{pmatrix} v_M \\ \hline . \\ \hline . \\ \hline F_{\mathrm{ext}} \end{pmatrix}$$

Formulations

1. Differential-Algebraic

$$\begin{pmatrix} \frac{d\mathbf{x}}{dt} \\ \hline \mathbf{w} \\ \hline -\mathbf{y} \end{pmatrix} = S \cdot \begin{pmatrix} \nabla H(\mathbf{x}) \\ \hline \mathbf{z}(\mathbf{w}) \\ \hline \mathbf{u} \end{pmatrix}, \qquad S = -S^T$$

2. Differential (1 → 2) by solving algebraic part $w = W(\nabla H(x), u)$

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = (J - R)\nabla H(x) + Gu, & J = -J^T, \quad R = R^T \geq 0 \\ -y = -G^T \nabla H(x) + Du, & D = -D^T \end{cases}$$

"Mass-Spring-Damper": $H(x) = \frac{x_1^2}{2M} + \frac{K x_2^2}{2}$, $z(w) = Cw$

$$S = \left[\begin{array}{cc|c|c} 0 & -1 & -1 & +1 \\ +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \\ \hline -1 & 0 & 0 & 0 \end{array}\right], \quad J = \begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix}, \; R = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix}, \; G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; D = 0$$

Outline

Power-balanced numerical method

(see also [Lopes et al., IFAC-LHMNLC'2015])

Classical numerical schemes for $\frac{\mathrm{d}x}{\mathrm{d}t} = f(x)$:

- efficiently approximate $\frac{\mathrm{d}.}{\mathrm{d}t}$ and exploit f
- *a posteriori* analysis of stability

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A discrete power-balanced method (PHS)

Exploit differentiation chain rule

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \sum_n \frac{\partial H}{\partial x_n}\frac{\mathrm{d}x_n}{\mathrm{d}t} \simeq \sum_n \underbrace{\frac{H_n(x_n[k+1]) - H_n(x_n[k])}{x_n[k+1] - x_n[k]}}_{\nabla^d H\left(x[k],\, \delta x[k]\right)_n} \underbrace{\frac{x_n[k+1] - x_n[k]}{\delta t}}_{(\delta x[k]/\delta t)_n} = \frac{E[k+1] - E[k]}{\delta t}$$

Jointly substitute $\dot{\mathbf{x}} \to \delta\mathbf{x}/\delta t$ and $\nabla H(\mathbf{x}) \to \nabla^d H(\mathbf{x}, \delta\mathbf{x})$:

$$\underbrace{\begin{pmatrix} \frac{\delta \mathbf{x}}{\delta t} \\ \mathbf{w} \\ -\mathbf{y} \end{pmatrix}}_{B} = S \underbrace{\begin{pmatrix} \nabla^d H(\mathbf{x}, \delta\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{pmatrix}}_{A}$$

Simulation : solve $(\delta\mathbf{x}, w)$ at each time step k (e.g. Newton-Raphson algo.)

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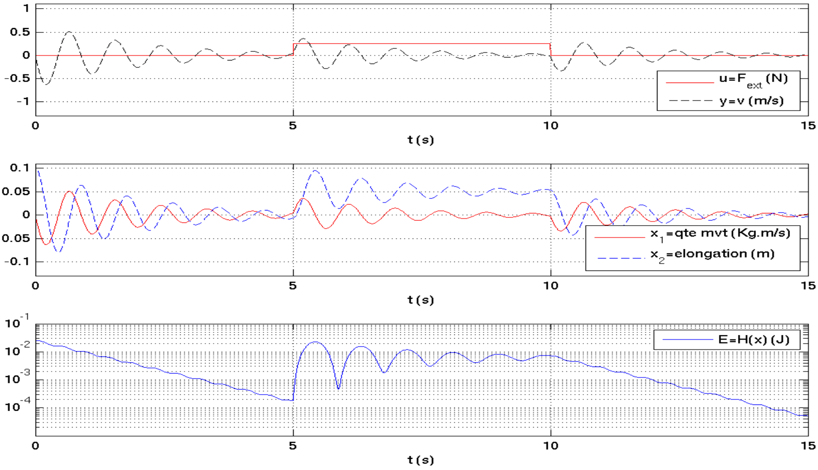
Simulation : solve $(\delta\mathbf{x}, w)$ at each time step k (e.g. Newton-Raphson algo.)

- Skew-symmetry of S preserved $\Rightarrow$ $0 = A^T S A = A^T B = \delta E/\delta t + \mathbf{z}(\mathbf{w})^T\mathbf{w} - \mathbf{u}^T\mathbf{y}$
- For **linear systems**, $\nabla^d H(\mathbf{x}, \delta\mathbf{x}) = \nabla(\mathbf{x} + \delta\mathbf{x}/2)$ restores the **mid-point scheme**.
- Method also applies to nonlinear components and non separate Hamiltonian
- Also available: Power-balanced Runge-Kutta scheme (non iterative algo.) [Lopes et al., LHMNLC'2015]

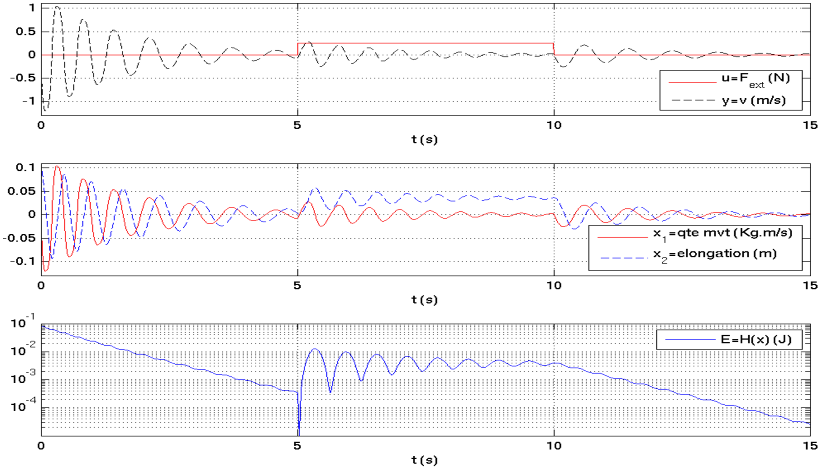
Simulation 1: mass-spring-damper

- **Parameters:** $M = 100\,\text{g}$, $K = 5\,\text{N/m}$, $C = 0.1\,\text{N.s/m}$ et $\delta t = 5\,\text{ms}$
- **Initial conditions:** $x_0 = \left[mv_0 = 0,\ \ell_0 = 10\,\text{cm} \right]^T$
- **Excitation:** $F_{\text{ext}}(t) = F_{\max}\ \mathbf{1}_{[5\text{s},10\text{s}]}(t)$ with $F_{\max} = K\ell_0/2 = 0.25\,\text{N}$



Simulation 2: idem with a hardening spring

- **Potential energy:** $H_2^{\mathrm{NL}}(x_2) = K\,L^2\left[\cosh(x_2/L) - 1\right] \quad \left(\sim k\,x_2^2/2\right)$
- **Physical law:** $F_2 = \left(H_2^{\mathrm{NL}}\right)'(x_2) = K\,L\,\sinh(x_2/L) \quad \left(\sim K\,x_2\right)$
- **Reference elongation:** $L = \ell_0/4 = 25\,\mathrm{mm}$



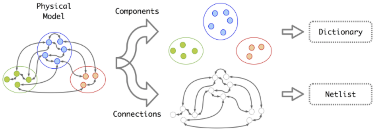
Outline

Automatic generation of code: the PyPHS Python library [Antoine Falaize]

https://pyphs.github.io/pyphs/

2012-16 : First version **[Falaize, PhD]**

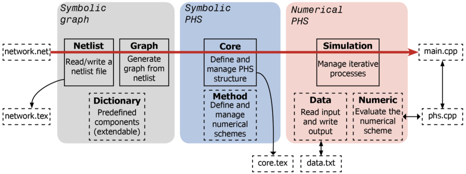
2016−− : Opensource library with periodic releases **[Falaize & contributors]**



The main objects of the library are introduced in this presentation. The standard workflow is as follows.

1. Inputs are **netlist descriptions** of network systems (very similar to SPICE netlists).
2. The associated **graphs** are analyzed to produce the **core system's dynamics equations** in the PHS formalism.
3. **Simulations** (i.e. numerical solving of DAE equations) are performed based on a variety of **numerical methods** (can be extended with new ones).
4. The corresponding **C++** simulation code is automatically generated and called from python (can also be used in bigger applications).
5. **LaTeX** description files can be generated (for documentation, publication, etc.).

Workflow

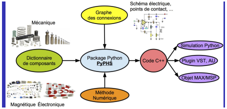


→ a short presentation (pdf file)

Outline

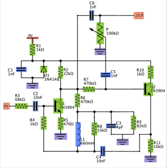
PhD, 2016: Antoine Falaize

Passive modelling, simulation, code generation
and correction of audio multi-physical systems



Two examples

Wah pedal (CryBaby): netlist → PyPHS → LateX eq. & C code



Components	Number
Storage	7 linear
Dissipative	18 (5 NL, 2 modulated)
Ports	3 (IN, OUT, battery)

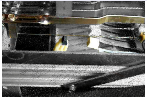
Audio PlugIn:

Sound 1a: dry

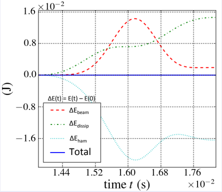
Sound 1b: wah

A simplified Fender-Rhodes Piano

Sound 2

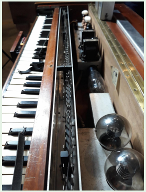


Components	Hammer	1 beam	Pickup/RC-circuit
Storage	2 NL	$2M$ lin.	2 lin. (+ NL connection)
Dissipative	1 NL	M lin.	1 lin.
Ports	2	1	1



Outline

Ondes Martenot (created by Maurice Martenot in 1928)

Controls	Circuit	*Diffuseurs*
 		

→ Video 3 [Thomas Bloch, improvisation, 2010]

Real-time simulation of Ondes Martenot

[Najnudel et al., AES2018]

Ondes Martenot (created by Maurice Martenot in 1928)

Controls

Circuit

Diffuseurs

→ Video 3 [Thomas Bloch, improvisation, 2010]

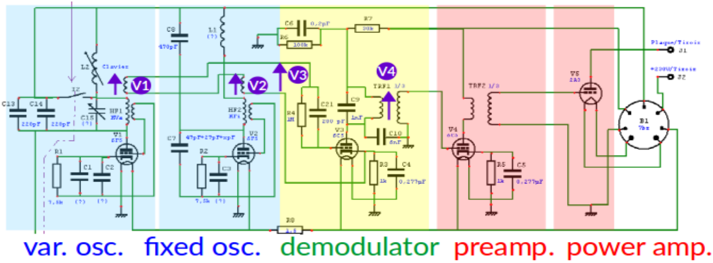
Context/Problem (Musée de la Musique, Philharmonie de Paris)

Technological obsolescence of a musical instrument:
70/281 remaining instruments *(handmade)*, **1200 pieces** *(Varèse, Maessian, etc)*

Objective (Collegium Musicae-Sorbonne Université)

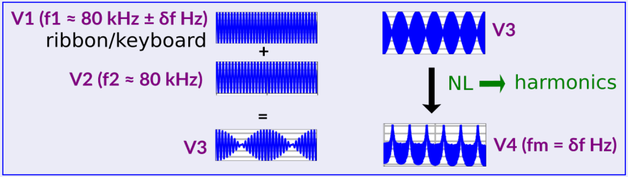
Real-time simulation of the circuit based on physics → PHS approach

Ondes Martenot: 5 stages circuit



Specificities: heterodyne oscillators (1930's)

- 2 High frequencies ($\approx$ 80kHz$\pm\delta f$) $\rightarrow$ demodulator $\rightarrow$ audio range ($\delta f, 2\delta f, \ldots$)

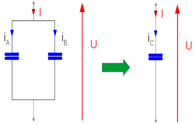


- Vacuum tubes: w = [grid and plate currents]^T, $z(w)$ = associated voltages

  (passive parametric model [Cohen'12])

- **Pb:** ribbon-controlled oscillator involving **time-varying capacitors in parallel**

Ondes Martenot: capacitors in parallel



Problem:

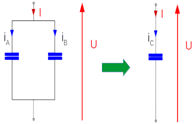
Capacitors	$(n = A, B)$
State *(charge)*:	q_n
Energy :	$H_n(q_n)$
Flux *(current)*:	$i_n = \mathrm{d}q_n/\mathrm{d}t$
Effort *(voltage)*:	$v_n = H_n'(q_n)$

$v_C = v_A = v_B$ &

$$\begin{bmatrix} i_A \\ i_B \\ -v_C \end{bmatrix} = \begin{bmatrix} \text{not} \\ \text{realizable} \end{bmatrix} \begin{bmatrix} v_A = H_A'(q_A) \\ v_B = H_B'(q_B) \\ i_C \end{bmatrix}$$

$\rightarrow$ Build the equivalent component $C = A//B$

Ondes Martenot: capacitors in parallel



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$\rightarrow$ Build the equivalent component $C = A//B$

Hyp: $q_n \mapsto v_n = H_n'(q_n)$ bijective (increasing law)

Find the total energy $H_C(q_C)$ for the total charge $q_C = q_A + q_B$

1. Charge as a function of the voltage $v_n = v_C$: $q_n = [H_n']^{-1}(v) := Q_n(v_C)$
2. Total charge (idem): $q_C = [Q_A + Q_B](v_C) =: Q_C(v_C)$
3. Total energy function: $H_C(q_C) = \sum_{n=A,B} H_n \circ Q_n \circ Q_C^{-1}(q_C)$

Also available if H_n depends on additional states (ribbon position ℓ)

Power-balanced simulation with $H(q, \ell) = q^2/(2C_{\mathrm{Martenot}}(\ell))$

$\rightarrow$ video 4 (sound=circuit output voltage, without the *diffuseurs*)

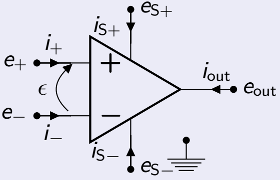
Outline

Operational Amplifier

[Muller et al., DAFx'19]

Idealized component

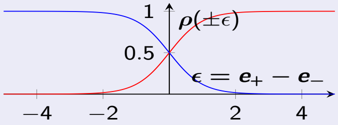
- **5 ports**



- **Algebraic conservative law**

$$\underbrace{\begin{bmatrix} i_+ \\ i_- \\ i_{S+} \\ i_{S-} \\ e_{out} \end{bmatrix}}_{z_{OPA}(w)} = \underbrace{\left[\begin{array}{cc|ccc} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \cdot & -\rho(+\epsilon) \\ \cdot & \cdot & \cdot & \cdot & -\rho(-\epsilon) \\ \cdot & \cdot & \rho(\epsilon) & \rho(-\epsilon) & \cdot \end{array}\right]}_{\mathbf{J}(w_{OPA}) = -\mathbf{J}(w_{OPA})^T} \underbrace{\begin{bmatrix} e_+ \\ e_- \\ e_{S+} \\ e_{S-} \\ i_{out} \end{bmatrix}}_{w_{OPA}}.$$

- **Modulation factor**

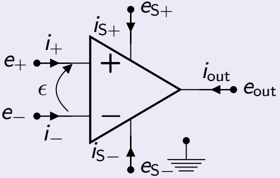


Operational Amplifier

[Muller et al., DAFx'19]

Idealized component

- **5 ports**



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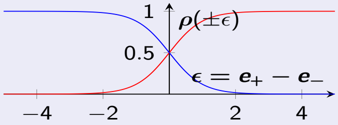
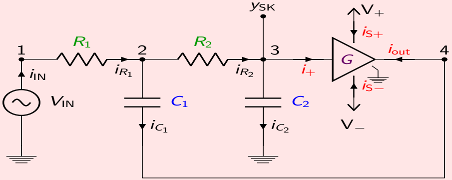
- **Modulation factor**



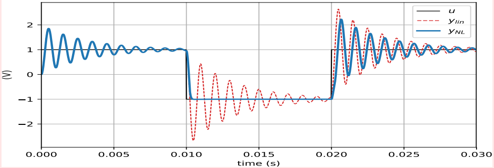
Typical analog filters (Sallen-Key)

- **Circuit:**



- **Nonlinear PHS:** $\begin{bmatrix} \dot{x} \\ w \\ w_{OPA} \\ -y \end{bmatrix} = S \begin{bmatrix} \nabla H(x) \\ z(w) \\ z_{OPA}(w_{OPA}) \\ u \end{bmatrix}$

- **Sounds 5 (simulations: linear / nonlinear)**



Outline

1. Context
2. **Framework:** basics, recalls and tools
3. **Analog electronics** and electro-acoustics
4. **Mechanics:** nonlinear damped vibrations
   - Nonlinear damping in a beam **[Hélie,Matignon,2015]**
   - Nonlinear Berger plate **[Hélie,Roze'18]**
   - Nonlinear string and Finite Element Method **[Raibeau, Roze'18]**
5. **Control** applications in acoustics
6. **Voice:** a minimal model to analyze self-oscillations
7. Conclusion

Motivation

1. Theoretical issues

Given a linear conservative mechanical system,

- find **damping models** that preserve the eigen modes (with eigen structure)
- design **nonlinear** damping in such a class
- provide a **power balanced formulation** that is preserved in **simulations**

2. Application in musical acoustics

Build **physical models** to produce:

- a **variety of beam sounds** (glokenspiel, xylophone, marimba, etc)
- **morphed sounds** through some extrapolations based on **physical grounds** *(e.g. meta-materials with damping depending on the magnitude)*

Damping models for $M\ddot{q} + C\dot{q} + Kq = f$ (finite-dimensional case)

Conservative problem (C=0)

- $\ddot{q} + (M^{-1}K)q = M^{-1}f$
- Eigen-modes e_i: $(M^{-1}K)e_i = \omega_i^2 e_i$ (ω_i: angular freq.)

Damping that preserves eigen-modes ?

- Choose $M^{-1}C$ as a **non-negative polynomial** of matrix $M^{-1}K$

$\rightarrow$ *Caughey class (1960)*: $C = c_0 M + c_1 K + c_2 K M^{-1} K + \dots$

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→ *Caughey class (1960)*: $C = c_0 M + c_1 K + c_2 K M^{-1} K + \dots$

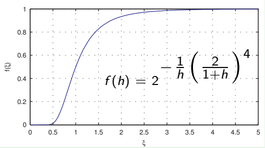
Eigen-modes with nonlinearly-damped dynamics ?

- Make c_n depend on the dynamics

Ex.: damping as a function of energy $H(x)$ (state $x = [q, p = M\dot{q}]^T$)

$c_n(x) = \kappa_n\big(H(x)\big) \in [c_n^-, c_n^+]$ with $c_n^- \geq 0$

- Increasing: $\kappa_n(h) = c_n^- + (c_n^+ - c_n^-) f(\frac{h}{h_0})$
- Decreasing: $\kappa_l(h) = c_n^+ - (c_n^+ - c_n^-) f(\frac{h}{h_0})$



Application: the Euler-Bernoulli beam

1. Pinned beam excited by a distributed force

(H1) Euler-Bernoulli kinematics: straight cross-section after deformation

(H2) linear approximation for the conservative problem

(H3) viscous and structural dampings: only $c_0, c_1 \geq 0$

2. Dimensionless model (w: deflection, $t \geq 0$, $0 \leq \ell \leq 1$)

- **PDE:** $\underbrace{\partial_t^2 w}_{\mathcal{M}\equiv Id} + \underbrace{(c_0 + c_1 \partial_\ell^4)}_{\mathcal{C}} \partial_t w + \underbrace{\partial_\ell^4}_{\mathcal{K}} w = f_{\mathrm{ext}}$
- **Boundaries** $\ell \in \{0, 1\}$**:** fixed extremities ($w = 0$), no momentum ($\partial_\ell^2 w = 0$)
- **Energy:** $E = \int_0^1 \left(\frac{(\partial_\ell^2 w)^2}{2} + \frac{(\partial_t w)^2}{2} \right) \mathrm{d}\ell$

3. Modal decomposition: $e_m(\ell) = \sqrt{2} \sin(m\pi\ell)$ $(1 \leq m \leq n)$

PHS:
$$\partial_t x = (J - R)\nabla H(x) + G u \quad \text{with} \quad J = \begin{bmatrix} 0_{n\times n} & I_n \\ -I_n & 0_{n\times n} \end{bmatrix}, \; R = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & C \end{bmatrix}$$
$$y = G^T \nabla H(X) \qquad G^T = [0_{n\times n}, I_n]$$

with $H\big(x = [q; p = M\dot{q}]\big) = \frac{1}{2} p^T M^{-1} p + \frac{1}{2} q^T K q$

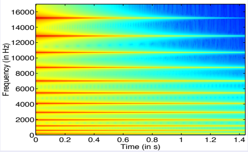
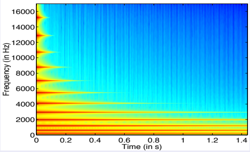
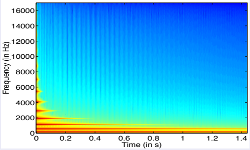
and $q = [q_1, \ldots, q_n]^T$, $u = [u_1, \ldots, u_n]^T$, $y = [y_1, \ldots, y_n]^T$

(projections of w, f_{ext}, v_{ext})

where $M = I_n$, $K = \pi^4 \mathrm{diag}(1, \ldots, n)^4$ and $C = c_0 I_n + c_1 K$.

Damping and simulation parameters

Examples of spectrograms for standard linear dampings: $c_0 \sim 10^{-2}$

metal ($c_1 \sim 10^{-6}$)	glass ($c_1 \sim 10^{-5}$)	wood ($c_1 \sim 10^{-4}$)
		

Nonlinear damping (from metal to wood):

$C(x) = c_0(x)I + c_1(x)K$ with
$c_n(x) = \beta_n\big(H(x)\big) \in [c_n^-, c_n^+]$

metal	$c_0^- = 0.02$	$c_1^- = 10^{-6}$
wood	$c_0^+ = 0.04$	$c_1^+ = 10^{-4}$

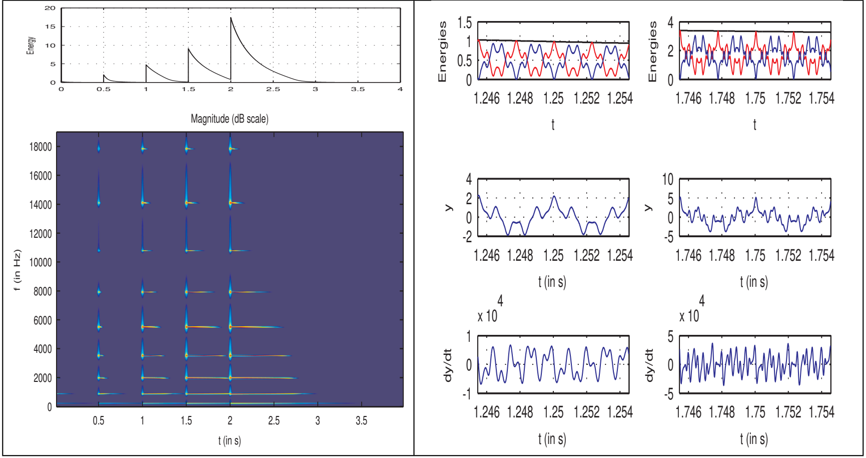
Numerical method preserving the power balance (discrete gradient)

- force distributed close to $z = 0$: $u = [1, \ldots, 1]^T f$
- listened signal: acceleration $[1, \ldots, 1]\dot{y}$
- $n = 9$ modes and time step s.t. $f_1 = 220\,\text{Hz}$ to $f_9 \approx n^2 f_1 = 17820\,\text{Hz}$

Results: $H(x) \ll 1 \longrightarrow$ wood, $H(x) \gg 1 \longrightarrow$ metal

force: 5 piecewise constant pulses (0.1ms) with increasing magnitude →Sound 6

overview	zooms total (black), kinetic (red) and potential (blue)



Outline

Nonlinear Berger plate

[Hélie,Roze'18]

1. Dimensionless model $(x, y) \in, \Omega = (0,1)^2,\ t \geq 0$

- PDE: $\partial_t^2 w + \alpha \partial_t w + \Delta\Delta w - \epsilon\left(\int_\Omega |\nabla w|^2 \mathrm{d}S\right)\Delta w = f$
- Zero IC & pinned BC ($w = 0$ on $\partial\Omega$, $\partial_x^2 w = 0$ if $x \in \{0,1\}$, $\partial_y^2 w = 0$ if $y \in \{0,1\}$)

Energy: $E = \int_\Omega \frac{(\partial_t^2 w)^2}{2} \mathrm{dS} + \int_\Omega \frac{(\Delta w)^2}{2} \mathrm{dS} + \epsilon \left(\int_\Omega \frac{(\nabla w)^2}{2} \mathrm{dS}\right)^2$

2. Modal decomposition (if $\epsilon = 0$): $e_{kl}(x,y) = 2\sin(k\pi x)\sin(l\pi y)$ $(1 \leq k, l \leq n)$

PHS: $\partial_t x = (J - R)\nabla H(x) + Gu$ with $J = \begin{bmatrix} 0_{n\times n} & I_n \\ -I_n & 0_{n\times n} \end{bmatrix}$, $R = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & C \end{bmatrix}$

$y = G^T \nabla H(X)$ $\quad G^T = [0_{n\times n}, I_n]$

$\rightarrow$ idem that the previous problem with

$K = \pi^4 \mathrm{diag}(1, \ldots, (k^2 + l^2)^2, \ldots, (n^2 + n^2)^2)$ and $C = \alpha I_n$

Nonlinear Berger plate

[Hélie,Roze'18]

1. Dimensionless model $(x,y) \in, \Omega = (0,1)^2,\ t \geq 0$

- PDE: $\partial_t^2 w + \alpha \partial_t w + \Delta\Delta w - \epsilon\left(\int_\Omega |\nabla w|^2 \mathrm{d}S\right)\Delta w = f$
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Nonlinear case: $\epsilon > 0$

- Modal decomposition available: $\epsilon\left(\int_\Omega |\nabla w|^2 \mathrm{d}S\right)\Delta w$ colinear to Δw
- Replace quadratic H by $H(q,p) = \frac{1}{2}p^T p + \frac{1}{2}q^T K q + \epsilon\left(\frac{1}{2}q^T K^{1/2} q\right)^2$

→ pitch effect on sounds→ video example 7 with sound and spectrum

Outline

Nonlinear Kirchhoff-Carrier String and Finite Element Method **[Raibeau,Roze'18]**

Nonlinear Kirchhoff-Carrier string model: PDE→FEM

$$\partial_t^2 w + \alpha \partial_t w - \left(1 + \varepsilon \int_0^1 (\partial_\ell w)^2 \, \mathrm{d}\ell \right) \partial_\ell^2 w = f$$

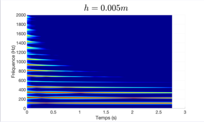
$$\xrightarrow{FEM} \mathbb{M}\ddot{W} + \mathbb{C}\dot{W} + (1 + \beta W^T \mathbb{K} W)\mathbb{K} W = F \quad \text{with} \quad W = [w_1 w_2 \ldots w_n]^T \, .$$

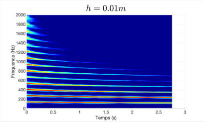
Energy: $H(Q = W, P = \mathbb{M}\dot{W}) = \frac{1}{2} P^T \mathbb{M}^{-1} P + \left(1 + \frac{\beta}{2} Q^T \mathbb{K} Q\right) \frac{1}{2} Q^T \mathbb{K} Q.$

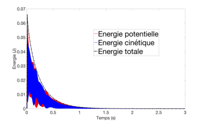
FEM→ PHS (Finite Elements Method in the PHS formalism)

$$\underbrace{\begin{bmatrix} \dot{Q} \\ \dot{P} \end{bmatrix}}_{\dot{X}} = \left(\underbrace{\begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}}_{J} - \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & \mathbb{C} \end{bmatrix}}_{R} \right) \underbrace{\begin{bmatrix} (1 + \beta Q^T \mathbb{K} Q)\mathbb{K} Q \\ \mathbb{M}^{-1} P \end{bmatrix}}_{\nabla H} + \underbrace{\begin{bmatrix} 0 \\ I \end{bmatrix}}_{G} \underbrace{\mathbb{F}}_{u}$$

Energy-preserving simulations of a nonlinear dynamics → sounds 8







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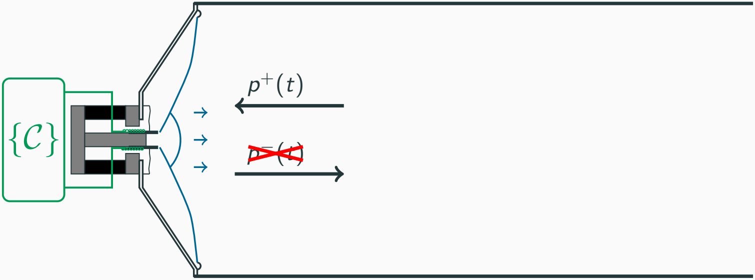
5. **Control** applications in acoustics
   - Passive Finite-Time Control of loudspeakers [Lebrun, Wijnand et al., Nodycon'19]
   - (zoom) Robotised testbed for brass instruments [Lopes,PhD'16]
6. **Voice:** a minimal model to analyze self-oscillations
7. Conclusion

Outline

Motivation: sound absorption



Here: {$\mathcal{C}$}

(a) Finite-time control ($\neq$ linear stabilization)

(b) Preserve controller passivity

Combination of (a) and (b): not straightforward.

Plane wave propagation in a tube

- **1D plane waves propagation**

$$p(z,t) = p^+(t - z/c_0) + p^-(t + z/c_0)$$

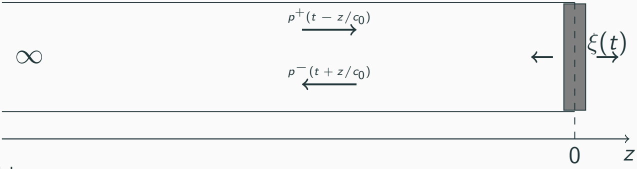
$$v(z,t) = \frac{p^+(t - z/c_0) - p^-(t + z/c_0)}{\rho_0 c_0}$$

- **Boundary condition at $z = 0$: rigid piston**

$$p^-(0,t) = p^+(0,t) - \rho_0 c_0 \dot{\xi}(t)$$

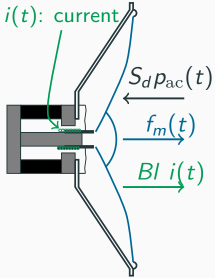
→ Impedance matching condition:

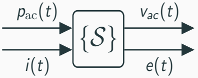
$$\dot{\xi}^\star(t) = \frac{p(0,t)}{\rho_0 c_0}$$



Current-driven loudspeaker

$$S_d p_{ac}(t) = \underbrace{M_m\ddot{\xi}(t) + R_m\ddot{\xi}(t) + K_m\xi(t)}_{f_m(t)} + Bl\ i(t)$$





- Laplace force
- Force due to the mechanical subsystem
- Force due to the acoustic pressure

Power-balanced formulation

Stored energy: $\mathcal{H}(\mathrm{x}) = K_m\frac{\xi^2}{2} + \frac{\mathfrak{p}^2}{2M_m} \geq 0$

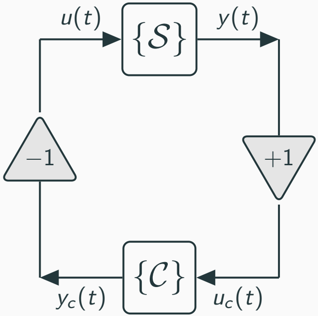
$$\underbrace{\begin{bmatrix}\dot{\xi}\\ \dot{\mathfrak{p}}\end{bmatrix}}_{\dot{\mathrm{x}}} = (\underbrace{\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}}_{\mathbf{J}=-\mathbf{J}^T} - \underbrace{\begin{bmatrix}0 & 0\\ 0 & R_m\end{bmatrix}}_{\mathbf{R}\geq 0}) \underbrace{\begin{bmatrix}K_m\xi\\ \mathfrak{p}/M_m\end{bmatrix}}_{\nabla\mathcal{H}(\mathrm{x})} + \underbrace{\begin{bmatrix}0 & 0\\ -Bl & S_d\end{bmatrix}}_{G} \underbrace{\begin{bmatrix}i\\ p_{ac}\end{bmatrix}}_{\mathrm{u}}$$

$$\underbrace{\begin{bmatrix}e\\ v_{ac}\end{bmatrix}}_{\mathrm{y}} = \underbrace{\begin{bmatrix}0 & -Bl\\ 0 & S_d\end{bmatrix}}_{G^T} \underbrace{\begin{bmatrix}K_m\xi\\ \mathfrak{p}/M_m\end{bmatrix}}_{\nabla\mathcal{H}(\mathrm{x})}$$

Port-Hamiltonian System [VanderShaft2014]

Ingredient 1: passive control (PHS approach)

A passive controller ?



- $\{\mathcal{S}\}$ is a port-Hamiltonian system (**PHS**):

$$\{\mathcal{S}\} \; : \begin{array}{l} \dot{\mathrm{x}}(t) = (\mathbf{J} - \mathbf{R})\,\nabla\mathcal{H}(\mathrm{x}) + \mathbf{G}\mathrm{u}(t) \\ \mathrm{y}(t) = \mathbf{G}^T\nabla\mathcal{H}(\mathrm{x}) \end{array}$$

- **Passivity:**

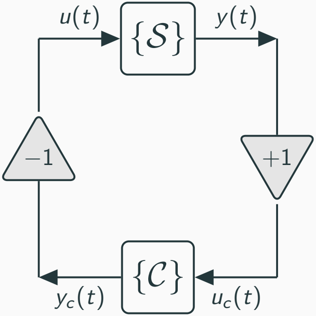
$$\underbrace{\nabla\mathcal{H}(\mathrm{x})^T\frac{\mathrm{dx}}{\mathrm{d}t}}_{\text{stored power}} \leq \underbrace{\mathrm{y}^T\mathrm{u}}_{\text{external power}}$$

- **Same property imposed to** $\{\mathcal{C}\}$ **:**

$$\{\mathcal{C}\} \; : \begin{array}{l} \dot{\mathrm{x}_c}(t) = (\mathbf{J}_c - \mathbf{R}_c)\,\nabla\mathcal{H}_c(\mathrm{x}_c) + \mathbf{G}_c\mathrm{u}(t \\ \mathrm{y}_c(t) = \mathbf{G}_c^T\nabla\mathcal{H}_c(\mathrm{x}_c) \end{array}$$

Ingredient 1: passive control

A passive controller ?



Interconnection of $\{\mathcal{S}\}$ & $\{\mathcal{C}\}$ **is a PHS**:

$$\begin{bmatrix} \dot{\mathrm{x}} \\ \dot{\mathrm{x}}_c \end{bmatrix} = \begin{bmatrix} \mathbf{J} - \mathbf{R} & -\mathbf{G}\mathbf{G}_c^T \\ \mathbf{G}_c\mathbf{G}^T & \mathbf{J}_c - \mathbf{R}_c \end{bmatrix} \nabla \mathcal{H}_{s+c}(\mathrm{x}, \mathrm{x}_c)$$

where $\mathcal{H}_{s+c}(\mathrm{x}, \mathrm{x}_c) = \mathcal{H}_s(\mathrm{x}) + \mathcal{H}_c(\mathrm{x}_c)$.

Example of control design

1. Define a total energy $\mathcal{H}_{s+c}(\mathrm{x}, \mathrm{x}_c)$ that has an equilibrium at a target $\mathrm{x}^\star$.
   $\mathcal{H}_{s+c}(\mathrm{x}, \mathrm{x}_c) = (\mathrm{x} - \mathrm{x}^\star)^2$.
2. Deduce the energy of the controller
   $\mathcal{H}_c(\mathrm{x}_c) = \mathcal{H}_{s+c}(\mathrm{x}, \mathrm{x}_c) - \mathcal{H}(\mathrm{x}) \geq 0$

Ingredient 2: finite-time control

Finite-time control of a double integrator [Bernuau2015]

Consider system

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = v \end{cases}$$

Using the control law for $0 < \alpha < 1$

$$v = -k_1 \lfloor z_1 \rceil^{\frac{\alpha}{2-\alpha}} - k_2 \lfloor z_2 \rceil^{\alpha}$$

with $\lfloor x \rceil^{\alpha} \triangleq \mathrm{sign}(x)|x|^{\alpha}$**,** $k_1 > 0$**,** $k_2 > 0$**, the origin is globally finite-time stable.**

- **Finite-time control $\rightarrow$ nonlinear control**
- **Reaches target in finite-time ($\neq$ asymptotic convergence)**
- × **Controller not passive**

Passive finite-time control

Application to the loudspeaker system

- Target closed-loop energy:

$$\mathcal{H}_{s+c}(\xi, \mathfrak{p}) = M_{\mathrm{m}} k_1 \frac{2-\alpha}{2} \left|\xi - \xi^\star\right|^{\frac{2}{2-\alpha}} + \frac{M_{\mathrm{m}} k_2}{R_{\mathrm{m}}} \frac{1}{\alpha+1} \left|\frac{\mathfrak{p} - \mathfrak{p}^\star}{M_{\mathrm{m}}}\right|^{\alpha+1}$$

$\rightarrow$ Minimum at $(\xi, \mathfrak{p}) = (\xi^\star, \mathfrak{p}^\star)$

$\rightarrow$ $\mathcal{H}_{s+c} - H \neq$ positive definite

Passive finite-time control

Application to the loudspeaker system

- Target closed-loop energy:

$$\mathcal{H}_{s+c}(\xi,\mathfrak{p}) = M_\mathrm{m} k_1 \frac{2-\alpha}{2} |\xi-\xi^\star|^{\frac{2}{2-\alpha}} + \frac{M_\mathrm{m} k_2}{R_\mathrm{m}} \frac{1}{\alpha+1} \left| \frac{\mathfrak{p}-\mathfrak{p}^\star}{M_\mathrm{m}} \right|^{\alpha+1} + \frac{\beta}{2}(\xi-\xi^\star)^2 + \frac{\gamma}{2M_\mathrm{m}}(\mathfrak{p}-\mathfrak{p}^\star)^2.$$

$\rightarrow$ Minimum at $(\xi,\mathfrak{p}) = (\xi^\star,\mathfrak{p}^\star)$

$\rightarrow$ $\beta > K_\mathrm{m}$ and $\gamma > R_m$ for **controller passivity**

- Energy of the controller: $\mathcal{H}_{s+c} - H_s = \mathcal{H}_c \geq 0$

$$\mathcal{H}_c(\xi,\mathfrak{p}) = M_m k_1 \frac{2-\alpha}{2} |\xi-\xi^\star|^{\frac{2}{2-\alpha}} + \frac{M_m k_2}{R_m} \frac{1}{\alpha+1} \left| \frac{\mathfrak{p}-\mathfrak{p}^\star}{M_m} \right|^{\alpha+1} + \frac{\beta}{2}(\xi-\xi^\star)^2 + \frac{\gamma}{2M_\mathrm{m}}(\mathfrak{p}-\mathfrak{p}^\star)^2 - \frac{1}{2M_m}\mathfrak{p}^2 - \frac{K_m}{2}\xi^2 \quad \geq 0$$

Passive finite-time control

Application to the loudspeaker system

- Controller $\{\mathcal{C}\}$:

$$\dot{\mathrm{x}} = \left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right) \nabla \mathcal{H}_c(\mathrm{x}) + \begin{bmatrix} \frac{1}{Bl} \\ \frac{R_\mathrm{m}}{Bl} \end{bmatrix} \mathrm{u}_c$$

$$\mathrm{y}_c = \begin{bmatrix} \frac{1}{Bl} & \frac{R_\mathrm{m}}{Bl} \end{bmatrix} \nabla \mathcal{H}(\mathrm{x}_c),$$

$\rightarrow \beta > K_\mathrm{m}$ and $\gamma > R_m$ for **controller passivity**

- Energy of the controller: $\mathcal{H}_{s+c} - H_s = \mathcal{H}_c \geq 0$

$$\mathcal{H}_c(\xi, \mathfrak{p}) = M_m k_1 \frac{2-\alpha}{2} |\xi - \xi^\star|^{\frac{2}{2-\alpha}} + \frac{M_m k_2}{R_m} \frac{1}{\alpha+1} \left| \frac{\mathfrak{p} - \mathfrak{p}^\star}{M_m} \right|^{\alpha+1}$$
$$+ \frac{\beta}{2}(\xi - \xi^\star)^2 + \frac{\gamma}{2M_\mathrm{m}} (\mathfrak{p} - \mathfrak{p}^\star)^2 - \frac{1}{2M_m}\mathfrak{p}^2 - \frac{K_m}{2}\xi^2 \quad \geq 0$$

Numerical results

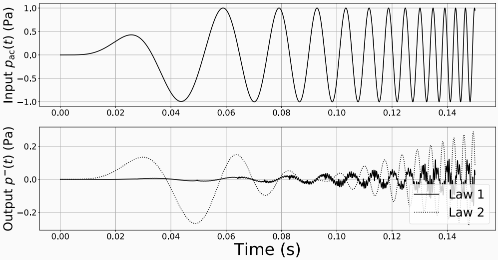
Simulation configuration

- Power-balanced numerical scheme
- Input: logarithmic chirp $p_{\mathrm{ac}}(t)$
- Output: reflected pressure $p^{-}(t)$

Test cases

- Law 1: proposed passive finite-time ($\rightarrow$ nonlinear) control
- Law 2: (reference) linear impedance-based control

Time domain simulation



Absorption $\geq 75\%$ in any case

Numerical results

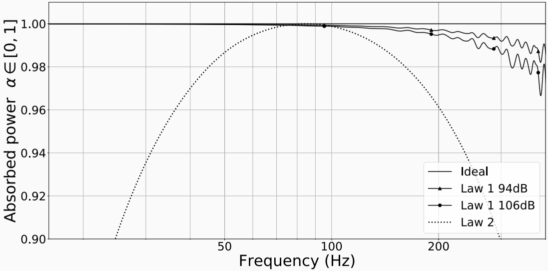
Simulation configuration

- Power-balanced numerical scheme
- Input: logarithmic chirp $p_{\mathrm{ac}}(t)$
- Output: reflected pressure $p^{-}(t)$

Test cases

- Law 1: proposed passive finite-time ($\rightarrow$ nonlinear) control
- Law 2: (reference) linear impedance-based control

Absorption versus frequency



Conclusion & perspectives

Contribution

- ✓ Construction and simulation of a passive finite-time control law
- ✓ Efficient sound absorption in the low-frequency audio range

Perspectives

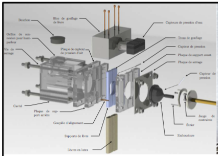
- Stiff problem of finite-time control around the origin (not Lipschitz continuous)
- Hardware implementation: passive delayed interconnection (current work)

Outline

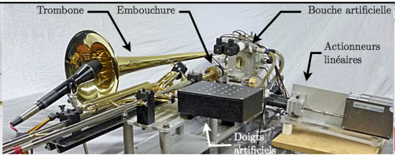
PhD, June 2016: Nicolas Lopes

Passive modelling, simulation and experimental study of a robotised artificial mouth playing brass instruments

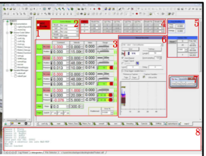
1. PHS/Simu of the complete system: *air jet in a channel with mobile walls, etc*



Instrumented mouth, lips and mouthpiece



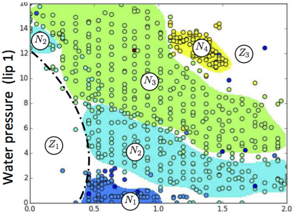
Robotised testbed



HMI *(dSpace & Python)*

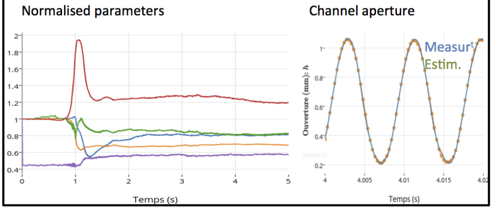
2. Automated exploration

Analysis of notes (N_j)



3. Estimation/Observation: extended Kalman filter on PHS

(5 estimated parameters: lip mass, stifness, *etc*)



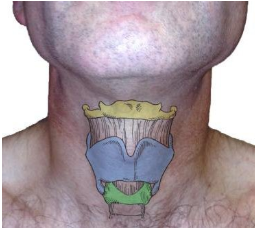
Outline

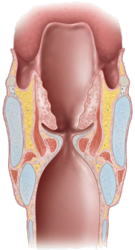
Vocal apparatus: Physiology & Physics

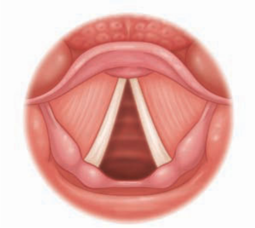
Air is forced out of the lungs . . . *(active)*

. . . at the top of the trachea, it goes through the glottis, *(passive)*
(=constriction between the vocal folds)

. . . before reaching the vocal tract *(=pharynx, nose, mouth)*. *(passive)*







Source: Seikel *et al*

Vocal apparatus: Physiology & Physics

Air is forced out of the lungs . . . *(active)*

. . . at the top of the trachea, it goes through the glottis, *(passive)*
(=constriction between the vocal folds)

. . . before reaching the vocal tract *(=pharynx, nose, mouth)*. *(passive)*

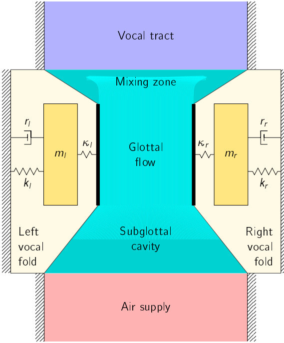
Phonation due to a **nonlinear** fluid/structure interaction in the larynx

Unstable equilibrium beyond some subglottal pressure threshold:

→ Vibration of the folds

→ Modulation of the glottal flow

→ Coupling with acoustic waves in the vocal tract



Most of state-of-the-art physics-based voice models are **not power-balanced** and **violate passivity**.

Vocal apparatus: Physiology & Physics

Air is forced out of the lungs . . . *(active)*

. . . at the top of the trachea, it goes through the glottis, *(passive)*

(=constriction between the vocal folds)

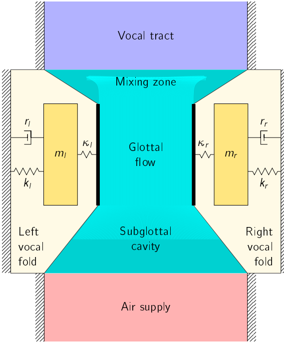
. . . before reaching the vocal tract *(=pharynx, nose, mouth)*. *(passive)*

Objective

Derive a **minimal model of the voice production** that

- restores a **power balance**
- is structured into components
- enables **power-balanced** time-domain simulations

and analyse its bifurcations and oscillation regimes



Outline

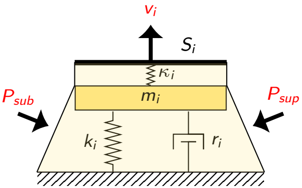
Vocal folds model $(i = l, r)$

Each fold is modelled as a spring-mass-damping system to be coupled to the glottal flow through a cover spring.

- Momentum π_i and elongations ξ_i and η_i

$$H = \frac{1}{2}\left(\frac{\pi_i^2}{m_i} + k_i\xi_i^2 + \kappa_i\eta_i^2\right)$$

- Dissipation $w_i = \pi_i/m_i$ and $z_i(w_i) = r_i w_i$
- Flow-controlled at the glottis port (v_i)
- Effort-controlled at the subglottal (P_{sub}) and supraglottal (P_{sup}) ports.



$$\begin{bmatrix} \dot{\xi}_i \\ \dot{\pi}_i \\ \dot{\eta}_i \\ \hline w_i \\ \hline -Q^i_{\mathrm{sub}} \\ Q^i_{\mathrm{sup}} \\ F_i \end{bmatrix} = \left[\begin{array}{ccc|c|ccc} & 1 & & & & & \\ -1 & & 1 & -1 & -S^i_{\mathrm{sub}} & -S^i_{\mathrm{sup}} & \\ & -1 & & & & & 1 \\ \hline & 1 & & & & & \\ \hline & S^i_{\mathrm{sub}} & & & & & \\ & S^i_{\mathrm{sup}} & & & & & \\ & & -1 & & & & \end{array}\right] \cdot \begin{bmatrix} \partial_{\xi_i} H \\ \partial_{\pi_i} H \\ \partial_{\eta_i} H \\ \hline z_i(w_i) \\ \hline P_{\mathrm{sub}} \\ P_{\mathrm{sup}} \\ v_i \end{bmatrix}$$

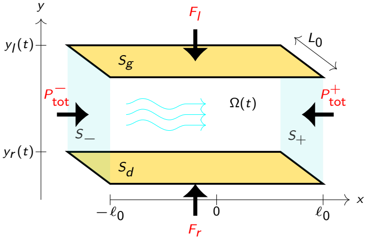
F_i Force of the vocal fold on the glottal flow

Q^i_{sup} Flowrate pushed into the supraglottal cavity

Q^i_{sub} Flowrate pulled from the subglottal cavity

Glottal flow between two parallel mobile walls

Objective: account for the transverse velocity and for power exchanged on the walls



Kinematics:

$$\mathbf{v} = \begin{pmatrix} V_x(t) \\ V_y(t) \end{pmatrix} + \frac{V_{exp}(t)}{h(t)} \begin{pmatrix} -x \\ y - y_m \end{pmatrix}$$

$V_x(t)$ mean axial velocity

$V_y(t) = \dot{y}_m$ mean transverse velocity

$V_{exp}(t) = \dot{h}$ transverse expansion velocity

$h(t)$ channel height

2D field: Euler equation

$$\nabla \cdot \mathbf{v} = 0, \quad \nabla \times \mathbf{v} = 0 \text{ on } \Omega(t)$$

$$\dot{\mathbf{v}} + \nabla \left(\frac{p}{\rho_0} + \frac{|\mathbf{v}|^2}{2} \right) = 0$$

BCs $\mathbf{v}_y(y_r) = \dot{y}_r \quad \mathbf{v}_y(y_l) = \dot{y}_l$

with $h = y_l - y_r$ et $y_m = \dfrac{y_l + y_r}{2}$

Reduced order dynamics

$$\begin{aligned} m(h)\dot{V}_x(t) &= L_0 h \left(P^-_{tot} - P^+_{tot}\right) \\ m(h)\dot{V}_y(t) &= F_r - F_l \\ m_e(h)\dot{V}_{exp}(t) &= L_0 \ell_0 \left(P^-_{tot} + P^+_{tot}\right) - \tfrac{F_r + F_l}{2} - \partial_h H \\ \dot{h} &= \tfrac{1}{m_e(h)} \partial_{V_{exp}} H \end{aligned}$$

$$H(V_x, V_y, V_{exp}, h) = \frac{1}{2} m(h) \left(V_x^2 + V_y^2\right) + \frac{1}{2} m_e(h) V_{exp}^2$$

Full vocal apparatus

Air supply: an ideal pressure supply P_{sub}

Subglottal cavity: uniform pressure/volume flow division

Vocal folds: as described before

Glottal flow: alternative formulation based on momentum (including scaling factor h_0)

Mixing region: the jet loses its kinetic energy into heat, without pressure recovery
Dissipation w_{mix} & uniform pressure/volume flow division

Vocal tract: modelled as an acoustic resonator,
with a modal representation of its input impedance.
For each resonance (ω_n, q_n, Z_n), state variables X_n^{ac} and Y_n^{ac} & dissipation w_n^{ac}

After elimination of linear dissipation variables:

$$
\begin{array}{c}
\text{Right vocal fold} \\ \hline \text{Left vocal fold} \\ \hline \text{Glottal flow} \\ \hline \text{Vocal tract} \\ \hline \text{Mixing} \\ \hline \text{Port}
\end{array}
\quad
\begin{pmatrix}
\dot{\xi}_r \\ \dot{\pi}_r \\ \dot{\eta}_r \\ \hline
\dot{\xi}_l \\ \dot{\pi}_l \\ \dot{\eta}_l \\ \hline
\dot{\Pi}_x \\ \dot{\Pi}_y \\ \dot{\Pi}_{exp} \\ \dot{h} \\ \hline
\dot{X}_1^{\text{ac}} \\ \dot{Y}_1^{\text{ac}} \\ \hline
w_{\text{mix}} \\ \hline
-Q_{\text{sub}}
\end{pmatrix}
=
\left(\begin{array}{ccc|ccc|cccc|cc|c|c}
 & 1 & & & & & & & & & & & & \\
-1 & -r_r & 1 & & & & & & & & -S^r_{\text{sup}} & & & -S^r_{\text{sub}} \\
 & -1 & & & & & & \frac{h_0}{h} & -1 & & & & & \\ \hline
 & & & & 1 & & & & & & & & & \\
 & & & -1 & -r_l & 1 & & & & & -S^l_{\text{sup}} & & & -S^l_{\text{sub}} \\
 & & & & -1 & & & -\frac{h_0}{h} & -1 & & & & & \\ \hline
 & & & & & & & & & & -L_0 h_0 & & -L_0 h_0 & L_0 h_0 \\
 & & -\frac{h_0}{h} & & & \frac{h_0}{h} & & & & & & & & \\
 & & 1 & & & 1 & & & & -2 & 2L_0\ell_0 & & 2L_0\ell_0 & 2L_0\ell_0 \\
 & & & & & & & & 2 & & & & & \\ \hline
 & S^r_{\text{sup}} & & & S^l_{\text{sup}} & & L_0 h_0 & & -2L_0\ell_0 & & -R_{\text{ac}} & -1 & & \\
 & & & & & & & & & & 1 & & & \\ \hline
 & & & & & & L_0 h_0 & & -2L_0\ell_0 & & & & & \\ \hline
 & S^r_{\text{sub}} & & & S^l_{\text{sub}} & & -L_0 h_0 & & -2L_0\ell_0 & & & & &
\end{array}\right)
\begin{pmatrix}
\partial_{\xi_r} H \\ \partial_{\pi_r} H \\ \partial_{\eta_r} H \\ \hline
\partial_{\xi_l} H \\ \partial_{\pi_l} H \\ \partial_{\eta_l} H \\ \hline
\partial_{\Pi_x} H \\ \partial_{\Pi_y} H \\ \partial_{\Pi_{exp}} H \\ \partial_h H \\ \hline
\nabla_{X_1^{\text{ac}}} H \\ \nabla_{Y_1^{\text{ac}}} H \\ \hline
z_{\text{mix}} \\ \hline
P_{\text{sub}}
\end{pmatrix}
$$

Outline

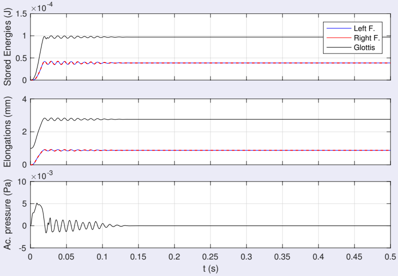
Power-balanced numerical experiments

Parameters values from literature:

Folds: $m_i = 0.2\,\mathrm{g}$, $r_i = 0.05\,\mathrm{N\,s\,m^{-1}}$, $\kappa_i = 3k_i$,
$S^i_{\mathrm{sup}} = 1.1\,\mathrm{mm}^2$, S^i_{sub}: $1.1\,\mathrm{cm}^2$
Glottis: $L_0 = 11\,\mathrm{mm}$, $2\ell_0 = 4\,\mathrm{mm}$
Vocal tract: first resonance of /a/
$f_0 = 640\,\mathrm{Hz}$, $q_0 = 2.5$, $Z_0 = 1\,\mathrm{M\Omega}$
Input P_{sub}: $0 \nearrow 800\,\mathrm{Pa}$ within $20\,\mathrm{ms}$

Open glottis ($h_r = 1\,\mathrm{mm}$)

$k_r = k_l = 100\,\mathrm{N\,m^{-1}}$
Natural frequency of folds: 112 Hz



No oscillation onset

Power-balanced numerical experiments

Parameters values from literature:

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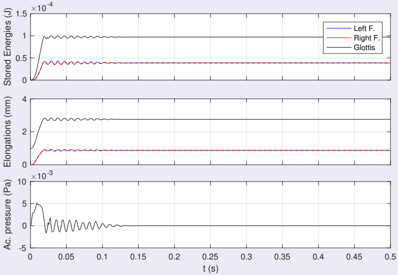
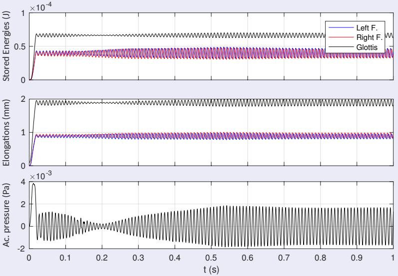


No oscillation onset

Adduction (nearly closed, $h_r = 0.1\,\mathrm{mm}$)

Slightly detuned vocal folds
$k_l = 100\,\mathrm{N\,m^{-1}}$ (112 Hz)
$k_r = 97\,\mathrm{N\,m^{-1}}$ (110 Hz)



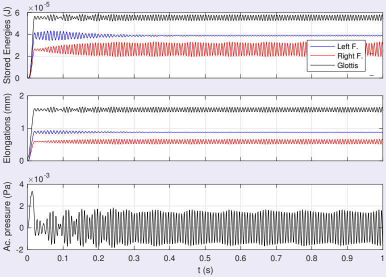
Periodic oscillations:

- Oscillation stabilized after some transient
- Synchronized folds vibrations even without contact between folds

Power-balanced numerical experiments

Strong asymmetry

$k_l = 100\,\mathrm{N\,m}^{-1}$ (112 Hz)
$k_r = 149\,\mathrm{N\,m}^{-1}$ (137 Hz)
with adduction ($h_r = 0.1\,\mathrm{mm}$)

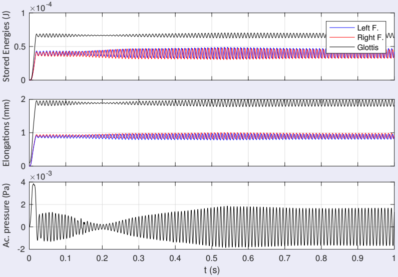


Quasi-periodic oscillations:

- starting on the left (lax) vocal fold,
- transferred to the right (stiffer) fold for the steady state regime.

Adduction (nearly closed, $h_r = 0.1\,\mathrm{mm}$)

Slightly detuned vocal folds
$k_l = 100\,\mathrm{N\,m}^{-1}$ (112 Hz)
$k_r = 97\,\mathrm{N\,m}^{-1}$ (110 Hz)

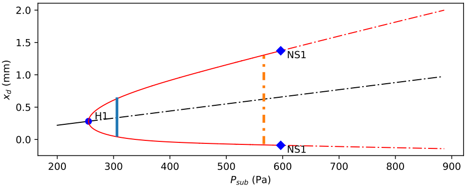


Periodic oscillations:

- Oscillation stabilized after some transient
- Synchronized folds vibrations even without contact between folds

Bifurcation analysis (internship L. Forma)

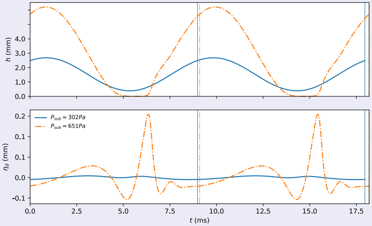
Coupling PyPHS with PyDSTool (continuation tool)



Hopf bifurcation

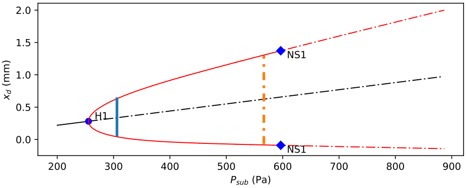
Onset pressure very close to the one obtained from time-domain simulation using PyPHS.

Beyond the Hopf bifurcation: limit cycles



Quasi sinusoidal oscillation near threshold

Coupling PyPHS with PyDSTool (continuation tool)



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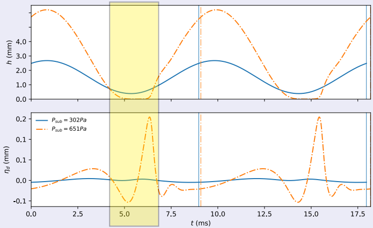
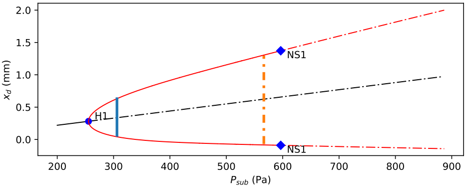


Force during glottal closure

Bifurcation analysis (internship L. Forma)

Coupling PyPHS with PyDSTool (continuation tool)



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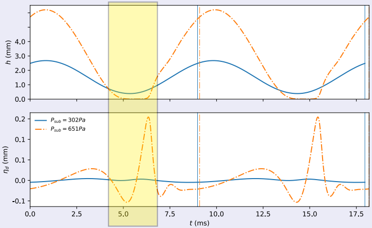
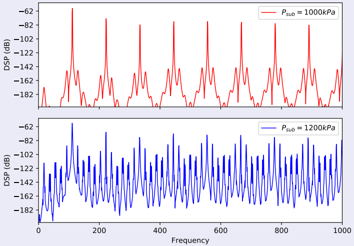


Force during glottal closure

Beyond the Neimark-Sacker bifurcation

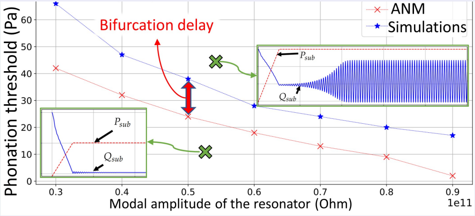


High pressure → raucous sound

Interpretation of other results on bifurcation analysis

Oscillation thresholds

- × Continuation of Hopf points (ANM)
- ⋆ Estimation from time-domain simulations

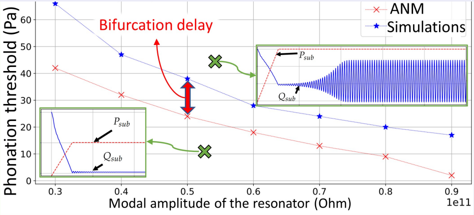


→ Explicit bifurcation delay
→ Related to dynamic bifurcation phenomenon?

Interpretation of other results on bifurcation analysis

Oscillation thresholds

- × Continuation of Hopf points (ANM)
- ⋆ Estimation from time-domain simulations

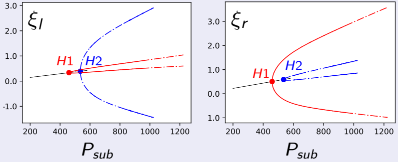


→ Explicit bifurcation delay
→ Related to dynamic bifurcation phenomenon?

Asymmetric vocal folds (L. Forma)

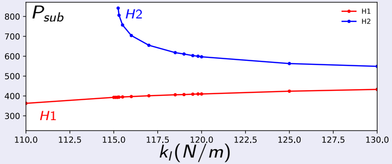
Bifurcation diagrams of ξ_l and ξ_r

$k_l = 150N/m$ $k_r = 100N/m$



Second Hopf bifurcation

for a sufficient degree of asymmetry

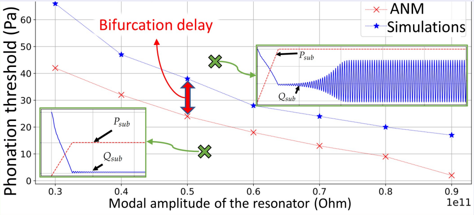


Interpretation of other results on bifurcation analysis

Oscillation thresholds

× Continuation of Hopf points (ANM)

⋆ Estimation from time-domain simulations

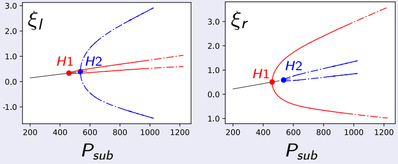


→ Explicit bifurcation delay
→ Related to dynamic bifurcation phenomenon?

Asymmetric vocal folds (L. Forma)

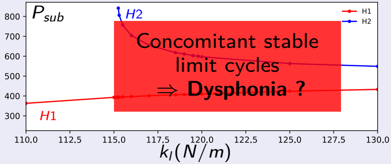
Bifurcation diagrams of ξ_l and ξ_r

$k_l = 150 N/m$ $k_r = 100 N/m$



Second Hopf bifurcation

for a sufficient degree of asymmetry



Outline

General conclusion and perspectives

Models and methods *(in progress)*

1. Nonlinear materials and damping **[Matignon, Roze]**
2. Various families of power-balanced numerical schemes **[Muller,Roze]**
3. Automatic code generators for real-time applications **[Falaize,Muller]**
4. Passive Finite-Time Control Design **[d'Andréa-Novel,Lebrun,Roze,Wijnand]**
5. Digital Passive Controller for hardware applications **[Lebrun]**
6. Voice physical-based synthesis **[Silva, Wetzel]**
7. Regime analysis of self-oscillating PHS **[Silva,Terrien,Wetzel]**

Projects based on PHS

1. Audio/Acoustics Virtual Factory
2. Augmented/Hybrid Musical Instruments with hardware development
3. Reprogrammed transducers (ideal HIFI louspeaker, acoustic absorber, etc)

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– The end –

Thank you for your attention